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ABSTRACT

Fourteen third graders were given numerical computation and division-with-remainder (DWR) problems both before and after they were taught the division algorithm in classrooms. Their solutions were examined. The results show that students' initial acquisition of the division algorithm did improve their performance in numerical division computations with small whole numbers but not in solving DWR problems. Students' acquisition of division algorithm led some of them to perceive a DRW problem as one that can be solved using division procedure. However, all of them retreated from their initial perception of division procedure to the execution of alternative procedures for solutions. The use of alternative procedures led these students to achieve similar success rate and treat a remainder in a similar way when solving DWR problems before and after learning division algorithm in classroom. (Author)

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DOES THE ACQUISITION OF MATHEMATICAL KNOWLEDGE MAKE STUDENTS BETTER PROBLEM SOLVERS? AN EXAMINATION OF THIRD GRADERS' SOLUTIONS OF DIVISION-WITH-REMAINDER (DWR) PROBLEMS

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Abstract: Fourteen third graders were given numerical computation and division-with-remainder (DWR) problems both before and after they were taught the division algorithm in classrooms. Their solutions were examined. The results show that students' initial acquisition of the division algorithm did improve their performance in numerical division computations with small whole numbers but not in solving DWR problems. Students' acquisition of division algorithm led some of them to perceive a DWR problem as one that can be solved using division procedure. However, all of them retreated from their initial perception of division procedure to the execution of alternative procedures for solutions. The use of alternative procedures led these students to achieve similar success rate and treat a "remainder" in a similar way when solving DWR problems before and after learning division algorithm in classrooms.

The development of students' proficiency in solving mathematics problems has been viewed as an important indicator of empowering students mathematically in school mathematics curriculum (e.g., National Council of Teachers of Mathematics [NCTM], 1989, 2000). Although it is generally believed that students' acquisition of formal mathematical knowledge can improve their competence in problem solving (e.g., Geary, 1995; Heffernan & Koedinger, 1997), previous studies have also shown that children and adults often use informal knowledge and strategies effectively in their problem-solving activities (e.g., Carraher, Carraher, & Schliemann, 1985; Scribner, 1984). Further explorations have indicated that the relationship between informal and formal knowledge can be either incongruous (e.g., Carraher, Carraher, & Schliemann ,1987) or complementary (Tabachneck, Koedinger, & Nathan, 1994; Vera & Simon, 1993). Although adequate use of formal knowledge can be much more mathematically powerful than the use of informal knowledge, informal knowledge can often help problem solvers in making sense of a mathematical problem and obtaining a solution. Different effects of using informal and formal knowledge suggest the need of further explorations on the role shift of informal and formal knowledge in students' problem-solving activities as they acquire more and more formal knowledge. Understanding such role changes can then provide a basis for informing classroom instruction. Along

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this direction, this study was designed to explore how students' acquisition of formal knowledge may change their uses of informal and formal knowledge in problem solving activities. In particular, this study explored how third graders' acquisition of division algorithm would affect their successes and sense-making behavior in solving Division-With-Remainder (DWR) problems.

Research Background

Middle school students' performance in solving DWR word problems in National Assessment of Educational Progress (1983) and several other studies (e.g., Silver, Shapiro, & Deutsch, 1993) exhibited that many middle school students solved a DWR problem by directly applying division procedure without interpretation of their calculation results based on the question being asked. It becomes clear in previous studies that many middle school students have no difficulty in understanding presented DWR problems and finding appropriate computation procedures for solution. Most errors happened in their final failure of interpreting computational results for answers (Silver et al., 1993). Missing link in connecting computational results and question asked at the final step indicated that many middle school students' solid acquisition of formal division algorithm led them to find an efficient solution strategy quickly but execute division procedure out of a problem context.

When a formal solution procedure was not ready accessible to students, a recent study on third graders' performance in solving a DWR word problem (Li & Silver, 2000) has illuminated students' super sense-making behavior with non-division solution strategies (e.g., counting up or down, multiplication). Although a DWR word problem was not the typical type of problems solved by third graders before their acquisitions of the division algorithm, third graders in this previous study showed their great success in solving a DWR problem. The contrast results between third graders and middle school students suggest the importance of understanding how the acquisition of formal division algorithm would likely affect students' sense-making behavior. Specifically, a study on the changes of third graders' success and sense-making behavior before and after their learning of division algorithm became feasible for pursuing this research inquiry.

Method

Fourteen third graders from a private laboratory school attached to a university participated in this study. This school is in an Eastern city. These children came from two classrooms taught by different teachers.

This study included two sections. The first section was carried out before these students' learning of division algorithm. The second section was given to the same group of third graders after their learning of division algorithm. These two sections were five month apart. In both sections, two types of problems were used. One type

was word problems with whole numbers. The others were numerical calculation problems. In this report, only the DWR word problems and numerical division computation problems were included for analyses (see them below).

Relevant problems used in section one (before students' learning of division algorithm):

DWR 1: Mary has 22 tapes. She wants to buy some boxes to store all her tapes. Each box can store 5 tapes. How many boxes does Mary need to buy?

Numerical division computations:

$20 \div 4 = 13\overline{)52}$

Relevant problems used in section two (after students' learning of division algorithm):

DWR 2.1: Tom has 28 tapes. He wants to buy some boxes to store all his tapes. Each box can store 6 tapes. How many boxes does Tom need to buy?

DWR 2.2: The Clearview Little League is going to a Pirates game. There are 540 people, including players, coaches, and parents. They will travel by bus, and each bus holds 40 people. How many buses will they need to get to the game?

Numerical division computations:

$30 \div 6 =$ $86 \div 12 =$ $252 \div 18 =$ $518 \div 30 =$

All tasks were administered to each student individually. Students were asked to think aloud when they solved the DWR problem(s), and their verbal explanations were recorded simultaneously for transcriptions. For the numerical division problems, students were asked to do computation on a piece of paper.

Students' solutions to numerical division items were coded as "correct", "incorrect" or "skipped". The transcriptions of students' verbal explanations of their solutions to the DWR word problems were analyzed both quantitatively and qualitatively. In particular, students' final answers were coded as correct or incorrect. Their solution process was subject to a fine-grained cognitive analysis. The cognitive analysis examined students' choice and execution of specific solution strategies in their solution processes. Four categories of solution strategies were developed and used in this analysis (taking students' solutions to DWR 1 as examples here):

1. Division (D). The student performed long division computation, 22÷5, to get the problem's solution.
2. Multiplication (M). The student used multiplication such as 4×5=20 and then figured out the number of boxes needed for 22 tapes to solve the problem.
3. Additive Approach (AA). The student used this approach (including subtraction) as counting up by 1's or 5's to 20 (or 25) and then counting the number of boxes needed for 22 tapes; or counting out 22 first, then counting out by 5's to figure out the number of boxes needed for 22 tapes.
4. Unidentifiable (U). The student either used some unknown strategies to solve the DWR problem or was unable to tell how he/she got the answer.

Results and Discussion

Quantitative Results

Table 1 shows the percentages of students obtaining correct numerical answers in solving the two types of problems. In section one, 43% students obtained a correct answer for "20 ÷ 4" and 29% for $13\overline{)52}$, whereas 93% of these solved the DWR 1 problem correctly. The results indicate that the lack of division procedure affected students' successes in numerical division computations but not in solving the DWR problem. In section two, 71% students did correct in calculating "30 ÷ 6", 21% for "86 ÷ 12", 7% for "252 ÷ 18", and none for "518 ÷ 30". 86% students obtained a correct answer in solving DWR 2.1, a problem containing small whole numbers, but only 29% in solving DWR 2.2, a problem with large numbers. The results suggest that students' initial acquisition of division algorithm helped students to perform well in solving both types of problems with small size of whole numbers but not in solving problems with large numbers. Across the two sections, the results show that students' acquisition of formal division algorithm did improve their performance in numerical division computations (e.g., 43% correct in calculating "20 ÷ 4" in section one versus 71% correct in calculating "30 ÷ 6" in section two) but not in solving the DWR problems (e.g., 93% correct in solving DWR1 in section one versus 86% correct in solving DWR2.1 in section two).

Table 1. Percentages of Students Obtained Correct Solutions in Two Sections.

Section One	20÷4	$13\overline{)52}$	DWR1			
	43%	29%	93%			
Section Two	30÷6	86÷12	252÷18	518÷30	DWR2.1	DWR2.2
	71%	21%	7%	0%	86%	29%

Qualitative Results

Table 2 shows the percentages of students who tended to choose a specific strategy after reading a DWR problem in the two sections. The results indicate that the Additive Approach was the most commonly chosen strategy by these students after reading these DWR problems in both sections (71% for DWR 1, 43% for DWR 2.1, and 71% for DWR 2.2). No student was able to perceive the DWR1 problem in section one as a problem that requires division procedure, whereas 36% of them chose division strategy at their first look at DWR2.1 in section two. Such difference indicates that initial acquisition of division algorithm led some students to perceive a DWR problem as one that requires a division procedure.

Table 3 shows the percentages of students who actually used a specific strategy to obtain a solution for these DWR problems in both sections. The results show that the Additive Approach also was the most commonly used strategy for solving these DWR problems (71% for DWR 1, 71% for DWR 2.1, and 79% for DWR 2.2). Although

Table 2. Percentages of Students Who Considered a Specific Solution Strategy After Reading the Problem During Solving DWR Problems in Two Sections.

		Division (D)	Multiplication (M)	Additive Approach (AA)	Unidentifiable (U)
Section One	DWR1	0%	29%	71%	0%
Section Two	DWR2.1	36%	21%	43%	0%
	DWR2.2	0%	21%	71%	7%

Table 3. Percentages of Students Who Actually Used a Specific Solution Strategy in Solving DWR Problems in Two Sections.

		Division (D)	Multiplication (M)	Additive Approach (AA)	Unidentifiable (U)
Section One	DWR1	0%	21%	71%	7%
Section Two	DWR2.1	0%	29%	71%	0%
	DWR2.2	0%	14%	79%	7%

some students (36%) perceived DWR problem 2.1 in section two as a problem that can be solved using division algorithm, none of them actually executed division procedure to obtain an answer. All of these students retreated from their initial choice of division procedure to the execution of other alternative procedures for solutions. Their uses of alternative procedures in solving DWR2.1 in section two and DWR1 in section one led them to achieve similar success rates across the two sections. These students' behavior suggests that their initial acquisition of division algorithm was not solid enough to help them to get the problem solution. The possible impact of division algorithm acquisition on students' problem understanding was evident but also limited as none of them perceived and solved the DWR2.2 as associated with the division algorithm. As a result, students' initial acquisition of division algorithm showed some impact on their understanding of a DWR problem but no apparent impact on their executions of solution procedures.

It is important to understand how students handled the remainder when studying possible changes in their sense-making behavior (Li & Silver, 2000; Silver et al., 1993). With their exclusive uses of non-division strategies for finding answers for all DWR problems, however, these students treated the remainder effectively in a similar way in both sections. Specifically, based on their situated reasoning and use of non-division strategies, these third graders treated a remainder concretely and successfully. For example, in solving DWR 1 in section one, these students took the remainder as "left over" or "extra" tapes or some other equivalent expressions in their solutions.

S: (After reading the problem) Five.

I: How do you know it is five?

S: Because if you bought four and each box can carry five tapes, that will be twenty tapes. And you need another one because there are *two more left*.

The results from this study indicate that students' initial acquisition of division procedure had limited impact on their performance in solving a DWR problem. Without a solid acquisition of the division algorithm, these third graders showed some 'division' sense when understanding a DWR problem but folded back to use non-division strategies in solving the DWR problem. Evidence exists that middle school students' sophisticated acquisition of division algorithm led them to perceive and solve a DWR problem using division procedure, but often disconnected from a problem context (Silver et al., 1993). Thus, students' acquisition of the division algorithm can have a great potential to enhance their computational efficiency, it can also change their sense-making behavior. The more sophisticated acquisition of division algorithm students have, the less use of alternative/informal strategies can be observed in their sense making and solving a mathematical problem. Therefore, in order to empower students with formal mathematical knowledge, it seems necessary for teachers to help students to make meaningful connections between formal mathematical knowledge

and various problem contexts. As there are different ways in classroom instruction that can help students to make such connections, it remains to be explored what impact different classroom instruction may have in facilitating students' learning and developing their problem-solving proficiency as expected.

Connections to the Goals of PME-NA

This study focused on the changes in students' sense-making behavior in solving mathematical problems before and after their learning of formal mathematics knowledge. It aims to provide a basis for understanding the role of formal mathematics knowledge in the development of students' mathematics competence and for generating pedagogical suggestions. Thus, this report connects to the goals of PME-NA in general and the coming PME-NA meeting in specific.

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